

# Allocative Efficiency and the Productivity Slowdown

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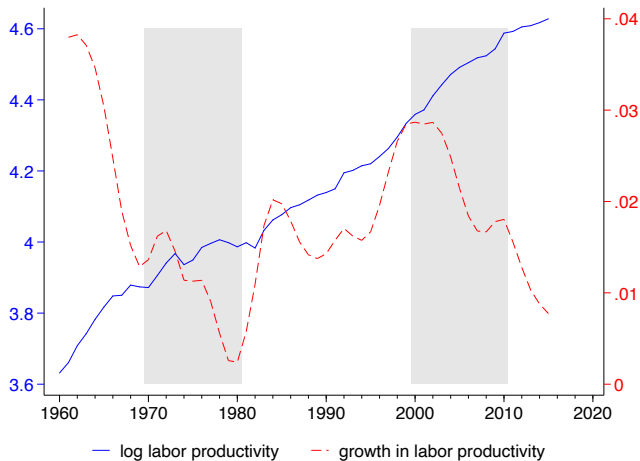
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The views expressed do not necessarily reflect the position of the Bank of Canada.

# Motivation

US labor productivity slowed down significantly during the 1970s and 2000s



# This paper

Evaluates the role of allocative efficiency across sectors in explaining aggregate productivity dynamics during these periods.

## Model

- Measure allocative efficiency in a multi-sector economy with or w/o input-output linkages.
- Decomposing productivity growth using sector-level data (KLEMS and WIOT).

## Findings

1. Allocation efficiency improved gradually during 1960-2010, contributing 20% of the observed productivity growth.
2. Deviations from this long-run trend helped give rise to slower-than-normal productivity growth during the 1970s and 2000s.
3. Crucial role of low volatility in the productivity process for allocation and growth.

# Model

# Production system

Value-added economy with Cobb-Douglas production functions

- One final good  $Y$ , produced by aggregating over  $N$  intermediate goods

$$Y_t = \prod_{i=1}^N Y_{i,t}^{\theta_{i,t}}.$$

- Intermediate good  $i$  is produced using capital and labor

$$Y_{i,t} = A_{i,t} K_{i,t}^{\alpha_{i,t}} L_{i,t}^{1-\alpha_{i,t}}.$$

# Optimal allocation

- Planner's problem

$$\max_{K_{i,t}, L_{i,t}} Y_t = \prod_{i=1}^N Y_{i,t}^{\theta_{i,t}}, \text{ s.t. } Y_{i,t} = A_{i,t} K_{i,t}^{\alpha_{i,t}} L_{i,t}^{1-\alpha_{i,t}}, \sum_i K_{i,t} = K_t, \sum_i L_{i,t} = L_t$$

- Optimal allocation:  $K_{i,t}^* = \chi_{i,t}^{k*} K_t$  and  $L_{i,t}^* = \chi_{i,t}^{l*} L_t$ .
  - $\chi_{i,t}^{k*} = \frac{\theta_{i,t} \alpha_{i,t}}{\sum_i \theta_{i,t} \alpha_{i,t}}$  and  $\chi_{i,t}^{l*} = \frac{\theta_{i,t} (1-\alpha_{i,t})}{\sum_i \theta_{i,t} (1-\alpha_{i,t})}$ .
  - $\chi_{i,t}^{k*}$  and  $\chi_{i,t}^{l*}$ : optimal sector-level shares of the aggregate capital and labor.

## Allocative efficiency

- Define allocative efficiency:  $E_t = \frac{Y_t}{Y_t^*}$ .
- Sufficient statistics for  $E_t$

$$E_t = \prod_{i=1}^N \left[ \left( \frac{\chi_{i,t}^k}{\chi_{i,t}^{k*}} \right)^{\alpha_{i,t}} \left( \frac{\chi_{i,t}^l}{\chi_{i,t}^{l*}} \right)^{1-\alpha_{i,t}} \right]^{\theta_{i,t}},$$

where  $\chi_{i,t}^k = \frac{K_{i,t}}{K_t}$  and  $\chi_{i,t}^l = \frac{L_{i,t}}{L_t}$  are cross-sector allocation of K and L in the **data**.

- Optimal allocation  $\Longleftrightarrow E_t = 1 \Longleftrightarrow \chi_{i,t}^k = \chi_{i,t}^{k*}$  and  $\chi_{i,t}^l = \chi_{i,t}^{l*} \forall i$ .

# Decomposition of aggregate productivity growth

- Decomposing the labor productivity growth in the data  $\log LP_t$  into:

$$\underbrace{\Delta \log LP_t}_{\text{productivity growth in the data}} = \underbrace{\Delta \log E_t}_{\text{growth in allocative efficiency}} + \underbrace{\Delta \log LP_t^*}_{\text{growth in fundamental productivity}} .$$



## Measuring allocative efficiency in the data

We need (i) allocation of capital and labor in the data and (ii) output elasticities ( $\alpha_{i,t}$ ,  $\theta_{i,t}$ )

**Data:** KLEMS (2013 version) with 28 private sectors. [▶ list of sectors](#)

- sector-level value-added output (\$).
- sector-level capital and labor compensation (\$).
- sector-level real capital stock, ( $K_{i,t}$ ) and number of workers ( $L_{i,t}$ ) (quantity)

## Measuring allocative efficiency in the data

**Identification issue:** Separately identify technology parameter  $\alpha_{i,t}$  and distortions  $(\tau_{i,t}^k, \tau_{i,t}^l)$ .

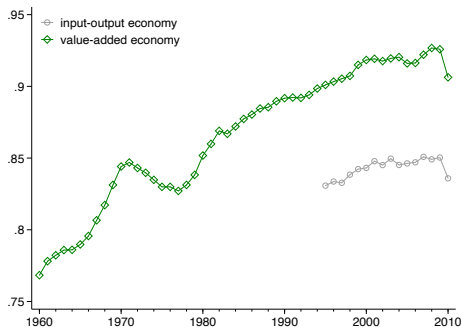
$$\frac{\overbrace{R_t K_{i,t}}^{\text{capital expenditure}}}{\underbrace{w_t L_{i,t}}_{\text{labor expenditure}}} = \frac{(1 - \tau_{i,t}^l) \alpha_{i,t}}{(1 - \tau_{i,t}^k)(1 - \alpha_{i,t})},$$

**Specifications:** Two approaches (assumptions) used in the literature.

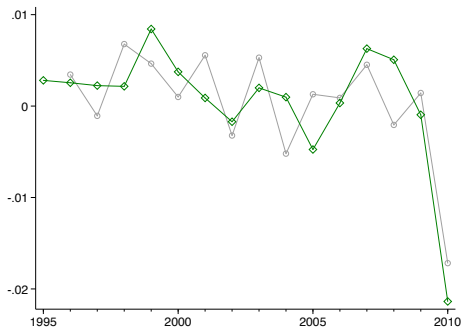
1. Use average labor and capital expenditure over a rolling window to identify  $\alpha_{i,t}$ .
  - Over time, distortions are, on average, not biased against K or L (Oberfield, 2013).
2. Use data of the later years to back out  $\alpha_i$ .
  - Economy is unconstrained in the later years (akin to Hsieh and Klenow, 2009).
  - Measured allocative efficiency is *relative to the base year*.

# Contribution of allocative efficiency to productivity slowdown

# Evolution of allocative efficiency over time



(a) Allocative efficiency



(b) Growth in allocative efficiency

- AE increases gradually, contributing to about 20 percent of the productivity growth.
- 1970s and 2000s are exceptions to this long-run trend.
- Measured AE is higher without input-output linkages, but growth rates are similar.

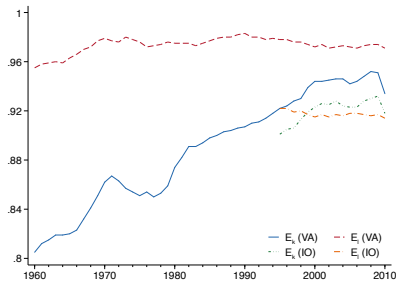
# Slowdown in productivity growth and the role of allocative efficiency

| Periods                           | Growth rates by periods<br>(long log-difference) |               |       | Changes in growth rates<br>from preceding period |               |              |
|-----------------------------------|--|---------------|-------|--|---------------|--------------|
|                                   | (1)  | (2)           | (3)   | (4)  | (5)           | (6)          |
|                                   | labor productivity                               |               |       | labor productivity                               |               |              |
|                                   | data   | "fundamental" | $E_t$ | data   | "fundamental" | $E_t$        |
| <b>(a) VA economy (1960–2007)</b> |  |               |       |  |               |              |
| 1960–69                           | 0.24   | 0.16          | 0.08  | –  | –             | –            |
| 1970–79                           | 0.13   | 0.13          | -0.01 | <b>-0.12</b>                                     | <b>-0.03</b>  | <b>-0.08</b> |
| 1980–89                           | 0.15   | 0.10          | 0.04  | 0.02   | -0.03         | 0.05         |
| 1990–99                           | 0.19   | 0.16          | 0.03  | 0.05   | 0.06          | -0.02        |
| 2000–07                           | 0.16   | 0.16          | 0.01  | <b>-0.03</b>                                     | <b>-0.01</b>  | <b>-0.02</b> |

- Labor productivity growth rate in the 1970s and 2000s was 12 pps and 3 pps lower than the preceding decades, respectively.
- Approximately two-thirds of the slowdown (8/12 and 2/3) was due to slow growth in allocative efficiency.

## Allocation efficiency of capital and labor

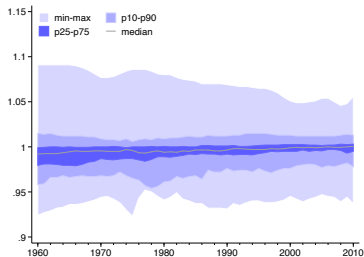
$$\mathbf{E}_t = E^{k,t} \times E^{l,t} = \underbrace{\prod_{i=1}^N \left( \frac{\chi_{i,t}^k}{\chi_{i,t}^{k*}} \right)^{\alpha_{i,t} \theta_{i,t}}}_{E^{k,t}} \times \underbrace{\prod_{i=1}^N \left( \frac{\chi_{i,t}^l}{\chi_{i,t}^{l*}} \right)^{(1-\alpha_{i,t}) \theta_{i,t}}}_{E^{l,t}}$$



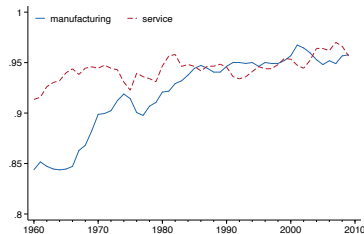
- Capital allocation is the more important driver across the sample period.

## Sector-level allocative efficiency

$$\mathbf{E}_t = \prod_{i=1}^N E_{i,t}^{\theta_{i,t}} = \prod_{i=1}^N \underbrace{\left[ \left( \frac{\chi_{i,t}^k}{\chi_{i,t}^{k*}} \right)^{\alpha_{i,t}} \left( \frac{\chi_{i,t}^l}{\chi_{i,t}^{l*}} \right)^{1-\alpha_{i,t}} \right]}_{E_{i,t}}^{\theta_{i,t}}$$



(a) Full distribution



(b) Manufacturing and service sector

- Distribution of  $E_i$  significantly narrows between 1960-70 and 1980-2000 but widens in the 1970s and stabilizes post-2000, mirroring the aggregate allocative efficiency trends.

## Recap of the results

- From 1960 to 2007, 20 percent of productivity growth came from improvements in allocative efficiency.
- This key driver of productivity growth was missing in the 1970s and 2000s, explaining 2/3 of the productivity slowdown.

### Results are robust:

CES production system; with input-output linkages; different start/end dates of the slowdown episodes; allocation within the manufacturing sector; post-2010 dynamics; heterogeneity in capital and labor inputs; allowing positive profit ...

### Next:

A potential driver behind the patterns of allocative efficiency during the two slowdown episodes.



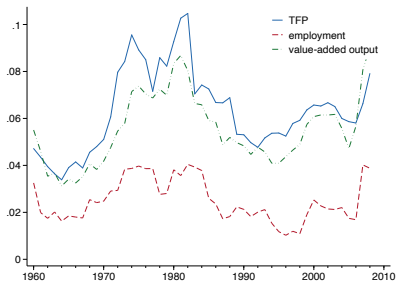
# Volatility as a potential driver

## Mechanisms:

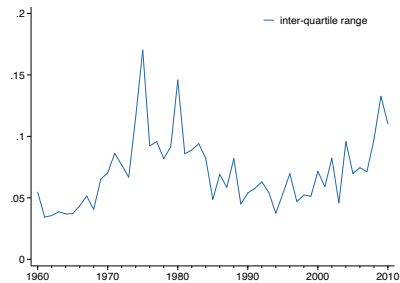
- With non-convex adjustment costs, there's an inaction region.
- As volatility increases
  - ⇒ option value of waiting increases.
  - ⇒ inaction region expands.
  - ⇒ sectors' capital and labor allocation are on average, further away from their optimal level.
  - ⇒ allocative efficiency declines.
- This mechanism was discussed in papers like Asker et al. (2014) and Bloom et al. (2018).

# Volatility and allocative efficiency during the slowdown episodes

## Sector-level shocks were more volatile during the 1970s and 2000s



(a) S.D. in sectoral growth rates



(b) Dispersion in TFP shocks

- The 1970s and 2000s experienced an increase in sector-level volatility.

## Relationship between volatility and allocative efficiency over time

|  | (1)                 | (2)                 | (3)                 |
|--|---------------------|---------------------|---------------------|
| Dispersion of TFP shocks in year $t$     | -0.072**<br>(0.031) | -0.087**<br>(0.037) | -0.080**<br>(0.037) |
| Dispersion of TFP shocks in year $t - 1$ |                     | 0.003<br>(0.043)    | 0.009<br>(0.046)    |
| Dispersion of TFP shocks in year $t - 2$ |                     |                     | -0.006<br>(0.027)   |
| Allocative efficiency in year $t - 1$    | 0.973***<br>(0.013) | 0.981***<br>(0.012) | 0.976***<br>(0.012) |
| Dependent variables                      | $\log(E_t)$         | $\log(E_t)$         | $\log(E_t)$         |
| N  | 63                  | 62                  | 61                  |
| $R^2$                                    | 0.993               | 0.993               | 0.993               |
| Observed slowdown in 1970s               | -0.12               | -0.12               | -0.12               |
| <b>Predicted slowdown in 1970s</b>       | <b>-0.10</b>        | <b>-0.10</b>        | <b>-0.10</b>        |
| Observed slowdown in 2000s               | -0.03               | -0.03               | -0.03               |
| <b>Predicted slowdown in 2000s</b>       | <b>-0.02</b>        | <b>-0.03</b>        | <b>-0.03</b>        |

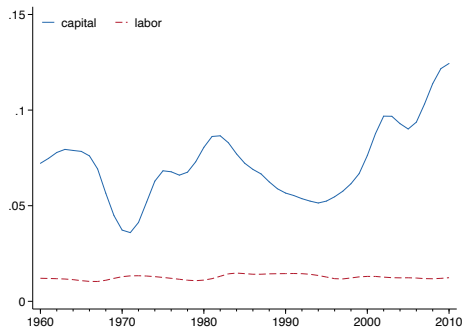
- Higher volatility is associated with lower allocative efficiency.
- Model predicts 10 pps and 2-3 pps decline in productivity growth for the 1970s and 2000s.

## Sector-level relationship between volatility and allocative efficiency

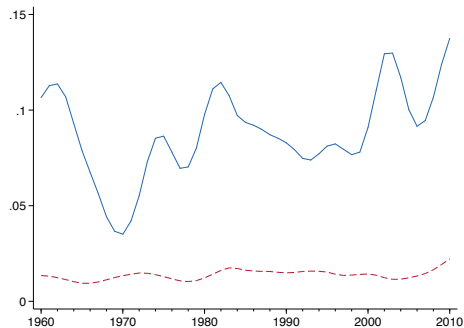
| Dependent variable $ \log E_{i,t}  -  \log E_{i,t-\Delta t} $ | $\Delta t = 2$      |                    |                    | $\Delta t = 4$     |                    |                   |
|---|---------------------|--------------------|--------------------|--------------------|--------------------|-------------------|
| Dispersion of TFP shock in $[t - \Delta t, t]$                | 0.141**<br>(0.0713) | 0.168*<br>(0.0866) | 0.154*<br>(0.0925) | 0.235**<br>(0.101) | 0.253**<br>(0.124) | 0.224*<br>(0.134) |
| Sector FEs  | N                   | Y                  | Y                  | N                  | Y                  | Y                 |
| Year FEs  | N                   | N                  | Y                  | N                  | N                  | Y                 |
| $N$   | 1593                | 1593               | 1593               | 1593               | 1593               | 1593              |
| $R^2$   | 0.021               | 0.060              | 0.104              | 0.029              | 0.081              | 0.127             |

- A rise in sector-level volatility is associated with reduced *sector-level allocative efficiency* ( $E_{i,t}$  deviates more from 1).

## Evidence from utilization rates



(a) Standard deviation



(b) Inter-quartile range

- In models with adjustment costs, factor utilization rates correspond to positions in the inaction regions (Abel and Eberly, 1998).

⇒ An expansion of the inaction region leads to increased dispersion of utilization rates.

# Conclusion

## Conclusion

- Allocative efficiency was a key factor behind both slow productivity growth episodes.
- Slower growth in allocative efficiency accounts for 2/3 of the productivity slowdown.
- Heightened volatility may have led to slow growth in allocative efficiency.

### Policy implications

- Critical role of volatility and allocative efficiency during periods of prolonged weak growth.
- This calls for policy measures to reduce volatility firms face or lower adjustment costs.



# Appendix

# List of sectors

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**AtB** Agriculture hunting forestry and fishing

**C** Mining and quarrying

**D** Manufacturing

15t16 Food products, beverages and tobacco

17t19 Textiles, textile products leather and footwear

20 Wood and products of wood and cork

21t22 Pulp paper, paper products, printing and publishing

23 Coke refined petroleum products and nuclear fuel

24 Chemicals and chemical products

25 Rubber and plastics products

26 Other non-metallic mineral products

27t28 Basic metals and fabricated metal products

29 Machinery nec

30t33 Electrical and optical equipment

34t35 Transport equipment

36t37 Manufacturing nec; recycling

**E** Electricity gas and water supply

**F** Construction

**G** Wholesale and retail trade

50 Wholesale trade and commission trade except of motor vehicles and motorcycles

51 Sale, maintenance and repair of motor vehicles and motorcycles; retail sale of fuel

52 Retail trade except of motor vehicles and motorcycles; repair of household goods

**H** Hotels and restaurants

**I** Transport and storage and communication

60t63 Transport and storage

64 Post and telecommunications

**J** Financial intermediation

**K** Real estate, renting and business activities

70 Real estate activities

71t74 Renting of m&eq and other business activities

**M** Education

**N** Health and social work

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# Input-output economy

## Production system

- One final good  $Y_t$

$$Y_t = \prod_{i=1}^N Y_{i,t}^{\theta_i}$$

- $N$  intermediate good  $\{Q_{i,t}\}_{i=1}^N$

$$Q_{i,t} = A_{i,t} (K_{i,t}^{\alpha_{i,t}} L_{i,t}^{1-\alpha_{i,t}})^{1-\sum_j^N \sigma_{ij,t}-\sum_j^N \lambda_{ij,t}} \left( \prod_{j,t}^N d_{ij,t}^{\sigma_{ij,t}} \right) \left( \prod_{j,t}^N m_{ij,t}^{\lambda_{ij,t}} \right)$$

- Capital:  $K_{i,t}$ . Labor:  $L_{i,t}$ .
  - Domestic intermediate goods from sector  $j$ :  $d_{ij,t}$ .
  - Imported intermediate goods from sector  $j$ :  $m_{ij,t}$ .
- Resource constraint on  $Q_{i,t} = Y_{i,t} + \sum_{j=1}^N d_{ji,t}$ .

# Input-output economy

## Allocative efficiency

- Sufficient statistics for  $E_t$

$$E_t = \frac{Y_t - X_t}{Y_t^* - X_t^*} = E_t^{kl} \cdot E_t^d \cdot E_t^m \cdot E_t^y,$$

- $E_t^{kl} = \prod_{i=1}^N \left( \left( \left( \frac{\chi_{i,t}^k}{\chi_{i,t}^{k*}} \right)^{\alpha_{i,t}} \left( \frac{\chi_{i,t}^l}{\chi_{i,t}^{l*}} \right)^{1-\alpha_{i,t}} \right)^{1-\sigma_{i,t}-\lambda_{i,t}} \right)^{\sum_n \theta_{n,t} C_{ni,t}}.$
- $E_t^d = \prod_{i=1}^N \left( \prod_{j=1}^N \left( \frac{\gamma_{ij,t}}{\gamma_{ij,t}^*} \right)^{\sigma_{ij,t}} \right)^{\sum_n \theta_{n,t} C_{ni,t}}.$
- $E_t^m = \frac{1 - \sum_{n=1}^N \frac{\theta_{n,t} \lambda_{n,t}}{\chi_{n,t}^y}}{1 - \sum_{n=1}^N \frac{\theta_{n,t} \lambda_{n,t}}{\chi_{n,t}^{y*}}}.$
- $E_t^y = \prod_{n=1}^N \left( \frac{\chi_{n,t}^y}{\chi_{n,t}^{y*}} \right)^{\theta_{n,t}} \prod_{i=1}^N \left( \frac{\prod_s \left( \frac{\chi_{s,t}^y}{\chi_{i,t}^{y*}} \right)^{\theta_{s,t}}}{\prod_s \left( \frac{\chi_{s,t}^y}{\chi_{i,t}^{y*}} \right)^{\theta_{s,t}}} \right)^{\lambda_{i,t} \sum_n (\theta_{n,t} C_{ni,t})}.$